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$\mathrm{Sp}(N)$ higher-derivative F-terms via singular superpotentials

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Abstract: We generalize the higher-derivative F-terms introduced by Beasley and Witten [1] for SU(2) superQCD to $\mathrm{Sp}(n_c)$ gauge theories with fundamental matter. We generate these terms by integrating out massive modes at tree level from an effective superpotential on the chiral ring of the microscopic theory. Though this superpotential is singular, its singularities are mild enough to permit the unambiguous identification of its minima, and gives sensible answers upon integrating out massive modes near any given minimum.

Keywords: Supersymmetric gauge theory, Supersymmetric Effective Theories.

Contents

1. Introduction

Supersymmetric gauge theories are more amenable to analysis than ordinary gauge theories (see [2, 3] for reviews). Until recently, much of the attention in supersymmetric gauge theories has been devoted to those with small numbers of flavors n_f. This is because for small n_f exact results, such as effective superpotentials, can easily be guessed and verified using some relatively simple consistency checks [4, 5]. This way of approaching the problem cannot easily be extended to larger numbers of flavors. Instead, one can reverse the strategy and start with the IR free regime of these theories where there are many massless flavors and strong quantum effects are negligible. The strong quantum effects found in the lower-flavor theories are obtained from higher-derivative F-terms of a special form in these IR free theories upon integrating out flavors [1].

These higher-derivative terms were calculated in [1] for SU(2) superQCD with $n_f \geq 2$ fundamental flavors using one-instanton methods. In [6] we computed these terms by integrating out massive modes at tree-level from an effective superpotential. These superpotentials are more singular than those normally considered: the potentials derived from them have cusp-like singularities at their minima. However, these singularities are mild enough that they unambiguously define the moduli space of vacua, and can be dealt with

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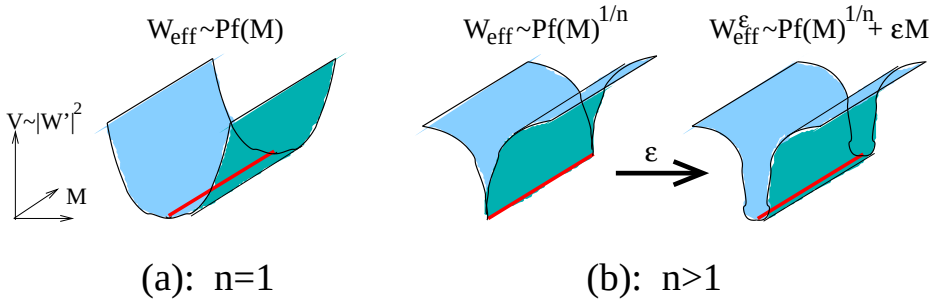


Figure 1: The effective potential as a function of the meson chiral fields M. The potential is regular for (a) $n = n_f - n_c - 1 = 1$, and singular for (b) $n > 1$ where the cusp-like singularity can be smoothed by a regularization parameter ε.

analytically by means of a simple regularization procedure. The intuitive picture [6] is summarized in figure 1. In [7] we also computed such superpotentials for SU(n_c) superQCD with $n_f = n_c + 2$.

In this paper we generalize these results, and the results of [1]. In section 2 we first compute the singular effective superpotentials of Sp(n_c) superQCD with matter in the fundamental representation, then we show that they correctly describe the moduli space of vacua, are consistent under RG flow to fewer flavors upon turning on masses, and are consistent solutions to the Konishi anomaly equations [8, 9]. Then, in section 3, we generalize the results of [1] to Sp(n_c) superQCD by expanding the superpotential around a generic vacuum and integrating out the massive modes of the meson field at tree level to find new higher-derivative F-terms.

2. Large n_f effective superpotentials of Sp(n_c) superQCD

2.1 Sp(n_c) superQCD for small n_f

Consider a four-dimensional $N = 1$ Sp(n_c) supersymmetric gauge theory with $2n_f$ massless quark chiral fields Q^i_a transforming in the fundamental representation, where $i = 1, \ldots, 2n_f$ and $a = 1, \ldots, 2n_c$ are flavor and color indices, respectively. (The number of flavors must be even for global anomaly cancellation [10].) The anomaly-free global symmetry of the theory is SU($2n_f$) $\times$ U(1)$_R$ under which the quarks transform as $(\mathbf{2n_f}, (n_f - n_c - 1)/n_f)$.

The classical moduli space of vacua is the space of vevs of holomorphic gauge-invariant chiral fields. For Sp(n_c) superQCD these are the anti-symmetric meson fields $\hat{M}^{[ij]} = Q^i_a J^{ab} Q^j_b$, where J^{ab} is the invariant antisymmetric tensor of Sp(n_c). (We distinguish vevs from operators by hatting operators.) For $n_f < n_c$ the classical moduli space is the space of arbitrary meson vevs M^{ij}, while for $n_f \geq n_c$ it is the set of all M^{ij} subject to the condition rank(M) $\leq 2n_c$, or equivalently

$$\epsilon_{i_1 \cdots i_{2n_f}} M^{i_1 i_2} \cdots M^{i_{2n_c+1} i_{2n_c+2}} = 0. \tag{2.1}$$

Quantum mechanically there is a dynamically generated superpotential [11]

$$W_{\text{eff}} = \begin{cases} (n_c+1-n_f)(\Lambda^{b_0}/\operatorname{Pf} M)^{1/(n_c+1-n_f)}, & \text{for } 0 < n_f \le n_c, \\ \Sigma\,(\operatorname{Pf} M - \Lambda^{b_0}), & \text{for } n_f = n_c+1, \\ -(\operatorname{Pf} M/\Lambda^{b_0}), & \text{for } n_f = n_c+2, \end{cases} \tag{2.2}$$

where the Pfaffian is defined as $\operatorname{Pf} M := \epsilon_{i_1\cdots i_{2n_f}} M^{i_1 i_2}\cdots M^{i_{2n_f-1} i_{2n_f}} = \sqrt{\det M}$, $b_0 = 3(n_c+1) - n_f$ is the coefficient of the one-loop beta function, Λ is the strong-coupling scale of the theory, and Σ is a Lagrange multiplier. These superpotentials encode the low energy behavior of the gauge theory: for $n_f \le n_c$ all the classical flat directions are lifted, for $n_f = n_c + 1$ instantons deform the classical moduli space, while for $n_f = n_c + 2$ the classical moduli space is not modified.

2.2 Superpotentials and classical constraints for large n_f

For $n_f > n_c+2$ the classical constraints are not modified, though there are new light degrees of freedom at singular subvarieties of the moduli space when the theory is asymptotically free, $n_f < 3n_c + 3$. These singular subvarieties are commonly refered to as the "origin" of the moduli space. The only effective superpotential (for points away from the origin) consistent with holomorphicity, weak-coupling limits, and the global symmetries is [11]

$$W_{\text{eff}} = -n\left(\frac{\operatorname{Pf} M}{\Lambda^{b_0}}\right)^{1/n}, \qquad n := n_f - n_c - 1 > 1. \tag{2.3}$$

The fractional power of $\operatorname{Pf} M$ implies that the potential derived from this superpotential has cusp-like singularities at its extrema. We will devote the rest of this section to arguing that, nevertheless, these singular superpotentials are physically perfectly sensible.

The first thing to check is to see whether these singular superpotentials describe the moduli space of vacua. Because these superpotentials are singular at their extrema we cannot just naively extremize them. We get around this problem by first deforming the superpotentials using some regularizing parameters ε_{ij}, extremizing them, then taking the $\varepsilon_{ij} \to 0$ limit at the end. Independent of how the regularizing parameters are sent to zero, the extrema of the superpotentials must reproduce the classical constraint (2.1). The superpotentials (2.3) indeed pass this check, as we now show.

We regularize (2.3) by adding a mass term with an invertible antisymmetric mass matrix ε_{ij} for the meson fields

$$W^{\varepsilon}_{\text{eff}} := W_{\text{eff}} + \frac{1}{2}\varepsilon_{ij} M^{ij}. \tag{2.4}$$

We have chosen to deform W_{eff} by a linear term in M^{ij} because it is simple, it smooths the singularity, and it does not dominate at large M, so does not introduce additional "spurious" extrema. We could have chosen a different deformation. Varying $W^{\varepsilon}_{\text{eff}}$ with respect to M^{kl} yields the equation of motion

$$M^{kl} = -\Lambda^{-b_0/n}(\operatorname{Pf} M)^{1/n}(\varepsilon^{-1})^{kl}. \tag{2.5}$$

Solving (2.5) for $\mathrm{Pf}\, M$ in terms of ε_{ij} and substituting back, we obtain

$$M^{kl} = -\Lambda^{-b_0/(n_c+1)} (\mathrm{Pf}\, \varepsilon)^{1/(n_c+1)} (\varepsilon^{-1})^{kl}. \tag{2.6}$$

Multiplying $n_c + 1$ copies of (2.6) together, and contracting the result with $\epsilon_{i_1 \ldots i_{2n_f}}$, we arrive at

$$\epsilon_{i_1 \ldots i_{2n_f}} M^{i_1 i_2} \cdots M^{i_{2n_c+1} i_{2n_c+2}} = \frac{(-1)^{n_c+1}}{\Lambda^{b_0}} \epsilon_{i_1 \ldots i_{2n_f}} (\varepsilon^{-1})^{i_1 i_2} \cdots (\varepsilon^{-1})^{i_{2n_c+1} i_{2n_c+2}}\, \mathrm{Pf}\, \varepsilon. \tag{2.7}$$

The right hand side of the above expression is a polynomial of order $n > 0$ in the ε_{ij}. Therefore, in the $\varepsilon_{ij} \to 0$ limit it vanishes independently of how we take the limit and we have

$$\epsilon_{i_1 \ldots i_{2N_f}} M^{i_1 i_2} \cdots M^{i_{2(N_c+1)-1} i_{2(N_c+1)}} = 0, \tag{2.8}$$

which is exactly the classical constraint that we wanted. Furthermore, it is easy to check that all solutions of the classical constraints can be reached by taking $\varepsilon_{ij} \to 0$ appropriately.

Note that the negative power of Λ appearing in (2.3) is not inconsistent with the weak coupling limit because the constraint equation (2.8) which follows from extremizing the singular superpotential implies that $\mathrm{Pf}\, M = 0$, thus W_{eff} vanishes on the moduli space for any finite value of Λ as well as in the $\Lambda \to 0$ limit.

We present another way of seeing how the classical constraints emerge from the singular superpotential which might make it clearer why these superpotentials have unambiguous extrema. Use the global symmetry to rotate the meson fields into the skew diagonal form

$$M^{ij} = \begin{pmatrix} M_1 & & & \\ & M_2 & & \\ & & \ddots & \\ & & & M_{n_f} \end{pmatrix} \otimes i\sigma_2, \tag{2.9}$$

so the effective superpotential (2.3) becomes $W_{\text{eff}} = -n\Lambda^{-b_0/n} (\prod_i M_i)^{1/n}$. The equations of motion which follow from extremizing with respect to the M_i are

$$M_i^{\frac{1}{n}-1} \prod_{j \neq i} M_j^{\frac{1}{n}} = 0. \tag{2.10}$$

Though these equations are ill-defined if we set any of the $M_i = 0$, we can probe the solutions by taking limits as some of the M_i approach zero. To test whether there is a limiting solution where K of the M_i vanish, consider the limit $\varepsilon \to 0$ with $M_1 \sim \varepsilon^{\alpha_1}, \ldots, M_K \sim \varepsilon^{\alpha_K}$ with $\alpha_j > 0$ to be determined. Note that different non-zero values α_j corresponds to different deformations in (2.4). Substituting into (2.10), only the first K equations have non-trivial limits,

$$\lim_{\varepsilon \to 0} \varepsilon^{\frac{1}{n}(\sum_j \alpha_j) - \alpha_i} = 0, \qquad i = 1, \ldots, K, \tag{2.11}$$

giving the system of inequalities $n\alpha_i < \sum_j \alpha_j$ for $i = 1, \ldots, K$. These inequalities have solutions if and only if $K > n$, implying that $\mathrm{rank}(M) \leq 2n_c$ which is precisely the classical constraint (2.1).

2.3 Consistency with Konishi anomaly equations: direct description

In the previous section the effective superpotential of the theory was determined by the global symmetry, weak-coupling limits, and holomorphicity. In this section we use the Konishi anomaly equations to derive the same superpotentials (2.3). The Konishi anomaly [8, 9] implies a set of differential equations which the effective superpotential should obey. We will show, using both the direct description and Seiberg dual description [12, 11] of the theory, that the solution to the Konishi anomaly equations coincides with our singular superpotentials. This is a consistency check on these superpotentials.

It has been shown [13] that for a pure superYang-Mills theory with the $\mathrm{Sp}(n_c)$ gauge group, the glueball superfield $\hat{S} = \frac{1}{32\pi^2}\,\mathrm{tr}(W^\alpha W_\alpha)$ generates all the local gauge-invariant chiral operators in the chiral ring of the theory. When we add matter multiplets to a superYang-Mills theory we also need to include local gauge-invariant matter generators. Following the arguments of [14, 15] it follows that $\hat{S}$ and $\hat{M}^{ij}$ comprise all the local gauge-invariant chiral generators in the chiral ring. In the chiral ring the Konishi anomaly for a tree level superpotential W_{tree} is

$$\left\langle \frac{\partial W_{\mathrm{tree}}}{\partial Q_a^i} Q_a^j \right\rangle = S\delta_i^j. \tag{2.12}$$

The above set of equations are perturbatively one-loop exact and do not get non-perturbative corrections. See [16, 6, 7] for discussions on the non-perturbative exactness of the Konishi anomaly equations.

As our tree level superpotential, we take

$$W_{\mathrm{tree}} = m_{ij}(\hat{M}^{ij} - M^{ij}). \tag{2.13}$$

where m_{ij} is a Lagrange multiplier enforcing $\hat{M}^{ij}$ to have M^{ij} as their vevs. It follows from the form of the above tree-level superpotential and the nature of the Legendre transform [2, 17, 18] that

$$m_{ij} = -\frac{1}{2}\frac{\partial W_{\mathrm{eff}}}{\partial M^{ij}}. \tag{2.14}$$

Substituting (2.13) into (2.12) and using the fact that the expectation value of a product of gauge-invariant chiral operators equals the product of the expectation values, gives $2m_{ik}M^{kj} = S\delta_i^j$. Using (2.14) we then obtain a set of partial differential equations for the effective superpotential

$$\frac{\partial W_{\mathrm{eff}}}{\partial M^{ik}} M^{kj} = S\delta_i^j, \tag{2.15}$$

whose solution is

$$W_{\mathrm{eff}}(M,S) = S\ln\left(\frac{\mathrm{Pf}\,M}{\Lambda^{n_f}}\right) + f(S), \tag{2.16}$$

where the stong-coupling scale of the theory Λ has been inserted to make the quantity inside logarithm dimensionless. The function $f(S)$ is determined by the $\mathrm{U}(1)_R$ symmetry to be

$$f(S) = -nS\left[\ln(S/\Lambda^3) - 1\right]. \tag{2.17}$$

(The constant term in the brackets, which can be absorbed in a re-definition of Λ, was determined by matching to the traditional normalization of the Veneziano-Yankielowicz superpotential [19] after giving masses and integrating out all the quarks.)

The glueball S is massive away from the origin so can be integrated out. Substituting (2.17) into (2.16) and integrating S out by solving its equation of motion, we arrive at the effective superpotentials (2.3).

2.4 Consistency with Konishi anomaly equations: Seiberg dual description

In this subsection we use the Konishi anomaly approach for the dual description as well as the Seiberg duality dictionary to rederive once again our singular effective superpotentials. For $n_f > n_c + 2$ the theory has a Seiberg dual description as an $\mathrm{Sp}(n_f - n_c - 2)$ supersymmetric gauge theory [11] with $2n_f$ (dual) quark chiral fields q_i^a, $i = 1 \ldots 2n_f$, in the fundamental representation and a gauge-singlet elementary field $\hat{\mathcal{M}}^{[ij]}$ coupled to the (dual) meson fields $\hat{\mathcal{N}}_{ij} := q_i^a J_{ab} q_j^b$ through the superpotential $W = \hat{\mathcal{N}}_{ij}\hat{\mathcal{M}}^{ij}$. The dual description is IR free when $n_f < \frac{3}{2}(n_c + 2)$.

The ring of local gauge-invariant chiral operators for the dual theory is generated by the dual glueball superfield $\hat{\mathcal{S}}$, $\hat{\mathcal{M}}^{ij}$ and $\hat{\mathcal{N}}_{ij}$. As our tree level superpotential we take

$$W_{\text{tree}} = \hat{\mathcal{N}}_{ij}\hat{\mathcal{M}}^{ij} + m_{ij}(\hat{\mathcal{M}}^{ij} - \mathcal{M}^{ij}), \tag{2.18}$$

where $m_{ij} = -\frac{1}{2}(\partial W_{\text{eff}}/\partial \mathcal{M}^{ij})$ is the Lagrange multiplier associated with the dual description (not to be confused with Lagrange multiplier of the direct description). It imposes the constraint that $\hat{\mathcal{M}}^{ij}$ have $\mathcal{M}^{ij}$ as their vevs. The superpotential $W = \hat{\mathcal{N}}_{ij}\hat{\mathcal{M}}^{ij}$ gives masses to the dual quarks and sets $\mathcal{N}_{ij} = 0$ when $\mathcal{M}^{ij} \neq 0$, which is why we have not included Lagrange multipliers for the dual mesons $\hat{\mathcal{N}}_{ij}$.

The Konishi anomaly equations for a tree level superpotential in the dual theory is $\langle(\partial W_{\text{tree}}/\partial q_i^a)q_j^a\rangle = \mathcal{S}\delta_j^i$. Substituting (2.18) gives $2\mathcal{M}^{ik}\mathcal{N}_{kj} = -\mathcal{S}\delta_j^i$. Using the $\hat{\mathcal{M}}^{ij}$ equation of motion, $\mathcal{N}_{ij} = -m_{ij}$, we can eliminate $\mathcal{N}_{kj}$ and arrive at

$$\mathcal{M}^{ik}\frac{\partial W_{\text{eff}}}{\partial \mathcal{M}^{kj}} = \mathcal{S}\delta_j^i, \tag{2.19}$$

whose solution is

$$W_{\text{eff}}(\mathcal{M}, S) = \mathcal{S}\ln\left(\frac{\mathrm{Pf}\,\mathcal{M}}{\tilde{\Lambda}^{n_f}}\right) + g(\mathcal{S}), \tag{2.20}$$

where $\tilde{\Lambda}$ is the strong-coupling scale of the dual theory. $g(\mathcal{S})$ is determined as before to be $g(\mathcal{S}) = -n\mathcal{S}[\ln(\mathcal{S}/\tilde{\Lambda}^3) - 1]$. Integrating out $\mathcal{S}$ then gives the effective superpotential in the dual description

$$W_{\text{eff}} = n\left(\tilde{\Lambda}^{3n-n_f}\,\mathrm{Pf}\,\mathcal{M}\right)^{1/n}. \tag{2.21}$$

The dual and the direct theories describe the same physics in the IR regime. Both theories have the same global symmetries and, away from the origin, they have the same moduli space and the same light degrees of freedom. They should also have the same effective superpotentials. Thus, relabeling (2.21) in terms of the direct theory degrees of freedom, we should recover the singular superpotential of the direct theory. In fact, using

the Seiberg duality dictionary, the $\mathcal{M}^{ij}$ are identified with the direct theory mesons through $\mathcal{M}^{ij} = \frac{1}{\mu} M^{ij}$, where μ is a mass scale related to the dual and the direct theory scales by

$$\Lambda^{3(n_c+1)-n_f} \tilde{\Lambda}^{3n-n_f} = (-1)^n \mu^{n_f}. \tag{2.22}$$

Using this, upon rewriting (2.21) in terms of Λ and M^{ij} we indeed find the direct theory superpotential (2.3).

2.5 Consistency upon integrating out flavors

Besides correctly describing the moduli space, the effective superpotentials should also pass some other tests. If we add a mass term for one flavor in the superpotentials of a theory with n_f flavors and then integrate it out, we should recover the superpotential of the theory with $n_f - 1$ flavors. To show that the effective superpotential (2.3) passes this test, we add a gauge-invariant mass term for one flavor, say $M^{2n_f-1\ 2n_f}$,

$$W_{\text{eff}} = -n\Lambda^{-b_0/n} (\text{Pf}\, M)^{1/n} + m M^{2n_f-1\ 2n_f}. \tag{2.23}$$

The equations of motion for $M^{i\ 2n_f-1}$ and $M^{j\ 2n_f}$ ($i \neq 2n_f - 1$ and $j \neq 2n_f$) put the meson matrix into the form $M^{ij} = \left(\begin{smallmatrix} \widehat{M} & 0 \\ 0 & \widehat{X} \end{smallmatrix}\right)$ where $\widehat{M}$ is a $2(n_f-1) \times 2(n_f-1)$ and $\widehat{X}$ a 2×2 matrix. Integrating out $\widehat{X} \sim M^{2n_f-1\ 2n_f}$ by its equation of motion gives

$$W_{\text{eff}} = -(n-1) \widehat{\Lambda}^{b_0/(n-1)} (\text{Pf}\, \widehat{M})^{1/(n-1)}, \tag{2.24}$$

where $\widehat{\Lambda} = m\Lambda^{3(n_c+1)-n_f}$ is the strong-coupling scale of the theory with $n_f - 1$ flavors, consistent with matching the RG flow of couplings at the scale m. Dropping the hats, we recognize (2.24) as the effective superpotentials of $\text{Sp}(n_c)$ superQCD with $n_f - 1$ flavors.

3. Higher-derivative F-terms in $\text{Sp}(n_c)$ superQCD

So far we have seen that our singular effective superpotentials (2.3) correctly describe the moduli space of vacua. In this section we will use these superpotentials to derive the form of certain higher-derivative F-terms in these theories. This derivation can be taken as a prediction for the result of instanton calculations in the $\text{Sp}(n_c)$ superQCD with large number of flavors.

In [1] Beasley and Witten showed that on the moduli space of SU(2) superQCD with $n_f \geq 2$, instantons generate a series of higher-derivative F-terms (also called multi-fermion F-terms). As F-terms they are protected by non-renormalization theorems, and so should be generated at tree level in perturbation theory from an exact low energy effective superpotential. Indeed, they also calculated these F-terms by integrating out massive modes at tree level from the non-singular effective superpotentials of SU(2) supersymmetic QCD with $n_f = 2$ and 3 flavors. In [6], it was shown that singular effective superpotentials of SU(2) supersymmetic QCD can reproduce the corresponding F-terms for $n_f > 3$, as well.

We will show in this section that the singular superpotentials of $\text{Sp}(n_c)$ superQCD (2.3) likewise generate higher-derivative F-terms by a tree-level calculation. As in our discussion

of the classical constraints in the last section, the key point in this calculation is to first regularize the effective superpotentials (2.3), and then show that the results are independent of the regularization.

The SU(2) F-terms of [1] have the form

$$\delta S = \int d^4x\, d^2\theta\, \Lambda^{6-n_f} (M\overline{M})^{-n_f} \epsilon^{i_1 j_1 \cdots i_{n_f} j_{n_f}} \overline{M}_{i_1 j_1} \times (M^{k_2 \ell_2} \overline{DM}_{i_2 k_2} \cdot \overline{DM}_{j_2 \ell_2}) \cdots (M^{k_{n_f} \ell_{n_f}} \overline{DM}_{i_{n_f} k_{n_f}} \cdot \overline{DM}_{j_{n_f} \ell_{n_f}}), \tag{3.1}$$

where $(M\overline{M}) := (1/2) \sum_{ij} M^{ij} \overline{M}_{ij}$, and the dot denotes contraction of the spinor indices on the covariant derivatives $\overline{D}_{\dot{\alpha}}$. Although these terms are written in terms of the unconstrained meson field, they are to be understood as being evaluated on the classical moduli space. In other words, we should expand the M^{ij} in (3.1) about a given point on the moduli space, satisfying $\epsilon_{i_1 \cdots i_{2n_f}} M^{i_1 i_2} \cdots M^{i_{2n_c+1} i_{2n_c+2}} = 0$, and keep only the massless modes (i.e. those tangent to the moduli space). We will refer to such terms as being "on vacuum" (in analogy to states being on mass-shell). They should be contrasted with our effective superpotentials (2.3) which are "off vacuum".

Even though (3.1) is written as an integral over a chiral half of superspace, it is not obvious that the integrand is a chiral superfield. But the form of the integrand is special: it is in fact chiral, and cannot be written as $\overline{D}^2$(something), at least globally on the moduli space, and so is a protected term in the low energy effective action [1].

3.1 Sp(n_c) F-terms

To derive on-vacuum effective interactions from an off-vacuum term, we simply have to expand around a given point on the moduli space and integrate out the massive modes at tree level; see figure 2. The only technical complication is that the effective superpotential needs to be regularized first, e.g. by turning on a small mass parameter ε_{ij} as in (2.4), so that it is smooth at its extrema. At the end, we take $\varepsilon_{ij} \to 0$. The absence of divergences as $\varepsilon_{ij} \to 0$ is another check of the consistency of our singular effective superpotentials. What we will actually compute is just the leading F-term in an expansion around a generic point on the vacuum in terms of the massless modes of the meson (those tangent to the moduli space).

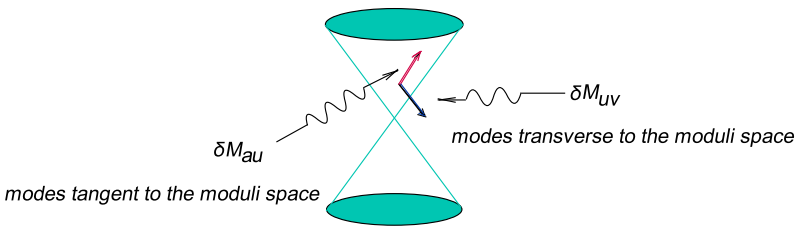


Figure 2: The massless tangent modes M_{au} (red arrow), and the massive transverse modes M_{uv} (blue arrow) after the meson field M_{ij} has been expanded around a given point on the moduli space.

As mentioned in section 2, the moduli space of vacua for $\mathrm{Sp}(n_c)$ superQCD with $n_f \geq n_c + 2$ flavors is given by the constraint

$$\mathrm{rank}(M^{ij}) \leq 2n_c. \tag{3.2}$$

At a generic point on the moduli space, the vev of the meson field ${M_0}^{ij} = \langle M^{ij} \rangle$ breaks the $\mathrm{SU}(2n_f)$ flavor symmetry. We can use flavor rotations to bring the generic vev into the form

$${M_0}^{ij} = \begin{pmatrix} \mu^{ab} & \\ & 0 \end{pmatrix}, \tag{3.3}$$

where $\mu^{ab} = \mu \mathbb{I}_{n_c} \otimes i\sigma_2$ is a skew-diagonalized antisymmetric matrix and μ is a complex parameter. Note that the above form for ${M_0}^{ij}$ breaks the $\mathrm{SU}(2n_f)$ flavor symmetry to $\mathrm{Sp}(2n_c) \times \mathrm{SU}(2n_f - 2n_c)$. Accordingly, we partition the $i, j, \ldots$ flavor indices into two sets: $\mathrm{Sp}(2n_c)$ indices $a, b, \ldots \in \{1, \ldots, 2n_c\}$ from the front of the alphabet, and $\mathrm{SU}(2n_f - 2n_c)$ indices $u, v, \ldots \in \{1, \ldots, 2n_f - 2n_c\}$ from the back. Linearizing the meson field around (3.3), $M^{ij} = {M_0}^{ij} + \delta M^{ij}$, subject to the constraint (3.2), implies that the massless modes are δM^{ab} and δM^{au}, while the δM^{uv} are all massive, as in figure 2. The δM^{ab} modes can be absorbed in a change of μ, so we only need to focus on the δM^{au} modes.

We will find that the leading F-term has the form

$$\begin{aligned} \delta S \sim \int d^4x \; d^2\theta \, \lambda^{-n} (\mu\overline{\mu})^{-n_f} \, \mathrm{Pf}(\overline{\mu}) \; \epsilon^{u_1 v_1 \cdots u_{n+1} v_{n+1}} \, (\mu^{c_1 d_1} \overline{D} \delta \overline{M}_{c_1 u_1} \cdot \overline{D} \delta \overline{M}_{d_1 v_1}) \times \\ \cdots \times (\mu^{c_{n+1} d_{n+1}} \overline{D} \delta \overline{M}_{c_{n+1} u_{n+1}} \cdot \overline{D} \delta \overline{M}_{d_{n+1} v_{n+1}}), \end{aligned} \tag{3.4}$$

where we have defined

$$n := n_f - n_c - 1, \qquad \text{and} \qquad \lambda := \Lambda^{-b_0/n} = \Lambda^{(n_f - 3n_c - 3)/n}. \tag{3.5}$$

Supersymmetry together with the flavor symmetry then uniquely determine the completion of this leading order term to all orders to be

$$\begin{aligned} \delta S = \int d^4x \; d^2\theta \, \Lambda^{3n_c + 3 - n_f} (M\overline{M})^{-n_f} \epsilon^{i_1 j_1 \cdots i_{n_f} j_{n_f}} \overline{M}_{i_1 j_1} \cdots \overline{M}_{i_{n_c} j_{n_c}} \\ \times (M^{k_{n_c+1} \ell_{n_c+1}} \overline{DM}_{i_{n_c+1} k_{n_c+1}} \cdot \overline{DM}_{j_{n_c+1} \ell_{n_c+1}}) \cdots (M^{k_{n_f} \ell_{n_f}} \overline{DM}_{i_{n_f} k_{n_f}} \cdot \overline{DM}_{j_{n_f} \ell_{n_f}}). \end{aligned} \tag{3.6}$$

This follows by an identical argument to one in [1]. Indeed, (3.6) is a straightforward generalization of (3.1) and has many similar properties, including that it is an F-term globally on the moduli space.

To generate the leading term (3.4), we first regularize $W_{\text{eff}} \to W_{\text{eff}}^{\varepsilon} = -n\lambda(\mathrm{Pf}\, M)^{1/n} + \frac{1}{2}\varepsilon_{ij} M^{ij}$, and choose $\varepsilon_{ij} = \lambda \varepsilon^{1/n} \mu^{2/n} \mathrm{diag}\{\varepsilon, 1, \ldots, 1\} \otimes i\sigma_2$ so that

$$(M_0^{\varepsilon})^{ij} = \begin{pmatrix} \mu^{ab} & \\ & \varepsilon^{uv} \end{pmatrix}, \tag{3.7}$$

where $\varepsilon^{uv} = \varepsilon \mathbb{I}_{n_f - n_c} \otimes i\sigma_2$ is a $2(n_f - n_c) \times 2(n_f - n_c)$ skew-diagonalized matrix. An advantage of this choice is that it preserves an $\mathrm{Sp}(2n_c) \times \mathrm{Sp}(2n_f - 2n_c)$ subgroup of the flavor symmetry. In the $\varepsilon \to 0$ limi, this symmetry is enhanced to $\mathrm{Sp}(2n_c) \times \mathrm{SU}(2n_f - 2n_c)$. Also, the massless directions around this choice of $(M_0^{\varepsilon})^{ij}$ are still δM^{ua} as before.

3.2 Feynman rules

We use standard superspace Feynman rules [20] to compute the effective action for the massless δM^{ua} modes by integrating out the massive δM^{uv} modes. This means we need to evaluate connected tree diagrams at zero momentum with internal massive propagators and external massless legs. In order to evalute these diagrams for the theory under discussion, we closely follow [6] where the superspace Feynman rules for SU(2) superQCD have been explained in detail. Generalizing these rules for $\mathrm{Sp}(n_c)$ superQCD is easy: the massive modes will have standard chiral, anti-chiral, and mixed superspace propagators with masses derived from the quadratic terms in the expansion of $W^{\varepsilon}_{\text{eff}}$, while higher-order terms in the expansion give chiral and anti-chiral vertices.

A quadratic term in the superpotential, $W = \frac{1}{2}m(\delta M)^2$, gives a mass which enters the chiral propagator as $\langle \delta M \delta M \rangle = \overline{m}(p^2 + |m|^2)^{-1}(D^2/p^2)$, similarly for the anti-chiral propagator, and as $\langle \delta M \delta \overline{M} \rangle = (p^2 + |m|^2)^{-1}$ for the mixed propagator. Each propagator comes with a factor of $\delta^4(\theta - \theta')$. Even though the diagrams will be evaluated at zero momentum, we must keep the p^2-dependence in the above propagators for two reasons. First, there are spurious poles at $p^2 = 0$ in the (anti-)chiral propagators which will always cancel against momentum dependence in the numerator coming from $\overline{D}^2$'s in the propagators and D^2's in the vertices. For instance, $D^2\overline{D}^2 = p^2$ when acting on an anti-chiral field, giving a factor of p^2 in the numerator which can cancel that in the denominator of the anti-chiral propagator, to give an IR-finite answer. Second, expanding the IR-finite parts in a power series in p^2 around $p^2 = 0$ can give potential higher-derivative terms in the effective action, when p^2's act on the external background fields. Expanding $W^{\varepsilon}_{\text{eff}}$ around $(M^{\varepsilon}_0)^{ij}$ gives the quadratic terms

$$W^{\varepsilon}_{\text{eff}}(M^{\varepsilon}_0 + \delta M) = W^{\varepsilon}_{\text{eff}}(M^{\varepsilon}_0) + \lambda\ (t_2)^{ijk\ell}_{i'j'k'\ell'}\ (\mathrm{Pf}\, M^{\varepsilon}_0)^{1/n}(M^{\varepsilon}_0)^{-1}_{ij}(M^{\varepsilon}_0)^{-1}_{k\ell}\delta M^{i'j'}\delta M^{k'\ell'} + \cdots, \tag{3.8}$$

where the numerical tensor $(t_2)^{ijk\ell}_{i'j'k'\ell'}$ controls how the $ij\ldots$ indices are contracted with the $i'j'\ldots$ indices. We will drop this tensor for now, though its form will be needed for a later argument. For our immediate purposes it suffices to note that in the $\varepsilon \to 0$ limit the tensor structure of our tree diagrams is fixed by the $\mathrm{Sp}(2n_c) \times \mathrm{SU}(2n_f - 2)$ subgroup of the global symmetry that is preserved by the vacuum.

Specializing to the massive modes, for which $\{i, j, k, \ell\} \to \{u, v, w, x\}$, and using (3.7) then gives the mass $m \sim \lambda\varepsilon^{-\alpha}\mu^{\beta}$, where

$$\alpha := \frac{n-1}{n}, \qquad \beta := \frac{n_c}{n}. \tag{3.9}$$

The propagators are then

$$\begin{aligned}
\delta M^{uv} \text{ - - - - } \delta M^{wx} \quad &\sim \quad \frac{\varepsilon^{\alpha}}{\lambda\mu^{\beta}}\frac{D^2}{p^2}\left(1 + \left|\frac{\varepsilon^{\alpha}}{\lambda\mu^{\beta}}\right|^2 p^2\right)^{-1}, \\
\delta\overline{M}_{uv} \text{ ———— } \delta\overline{M}_{wx} \quad &\sim \quad \frac{\overline{\varepsilon}^{\alpha}}{\overline{\lambda}\overline{\mu}^{\beta}}\frac{\overline{D}^2}{p^2}\left(1 + \left|\frac{\varepsilon^{\alpha}}{\lambda\mu^{\beta}}\right|^2 p^2\right)^{-1},
\end{aligned}$$

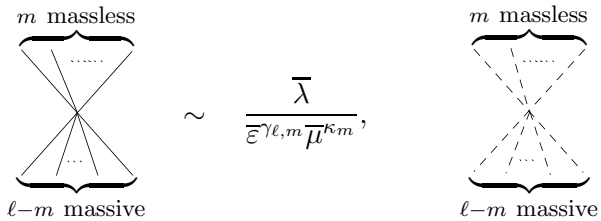
$$\delta\overline{M}_{uv} \text{ —— } - - \delta M^{wx} \quad \sim \quad \left|\frac{\varepsilon^\alpha}{\lambda\mu^\beta}\right|^2 \left(1 + \left|\frac{\varepsilon^\alpha}{\lambda\mu^\beta}\right|^2 p^2\right)^{-1}, \tag{3.10}$$

where have suppressed the tensor structures on the $\{u, v, w, x\}$ indices.

The (anti-)chiral vertices come from higher-order terms in the expansion of $W^\varepsilon_{\text{eff}}$ ($\overline{W^\varepsilon_{\text{eff}}}$). Each (anti-)chiral vertex will have a $\overline{D}^2$ (D^2) acting on all but one of its internal legs. Also, each vertex is accompanied by an $\int d^4\theta$. The ℓth-order term in the expansion of $W^\varepsilon_{\text{eff}}$ has the general structure

$$\lambda\ (t_\ell)^{i_1 j_1 \cdots i_\ell j_\ell}_{i'_1 j'_1 \cdots i'_\ell j'_\ell}\ (\text{Pf}\, M^\varepsilon_0)^{1/n} (M^\varepsilon_0)^{-1}_{i_1 j_1} \cdots (M^\varepsilon_0)^{-1}_{i_\ell j_\ell} \delta M^{i'_1 j'_1} \cdots \delta M^{i'_\ell j'_\ell}, \tag{3.11}$$

where the numerical tensor $(t_\ell)^{i_1 j_1 \cdots i_\ell j_\ell}_{i'_1 j'_1 \cdots i'_\ell j'_\ell}$ controls how the $i_1 j_1 \cdots i_\ell j_\ell$ indices are contracted with the $i'_1 j'_1 \cdots i'_\ell j'_\ell$ indices. Thus vertices with m massless legs and $\ell - m$ massive legs are accompanied by the factors



$$\sim \quad \frac{\overline{\lambda}}{\overline{\varepsilon}^{\gamma_{\ell,m}} \overline{\mu}^{\kappa_m}}, \qquad\qquad \sim \quad \frac{\lambda}{\varepsilon^{\gamma_{\ell,m}} \mu^{\kappa_m}}, \tag{3.12}$$

where

$$\gamma_{\ell,m} := \ell - \frac{m}{2} - \frac{n+1}{n}, \qquad\qquad \kappa_m := \frac{m}{2} - \frac{n_c}{n}. \tag{3.13}$$

Note that it follows from (3.11) that the number, m, of massless legs δM^{au} must be even. This is because these legs each have one index $a \in \{1, 2, \ldots 2n_c\}$ and the only non-vanishing components of $(M^\varepsilon_0)^{-1}_{ij}$ with indices in this range are $(M^\varepsilon_0)^{-1}_{ab} = -(M^\varepsilon_0)^{-1}_{ba} = \mu^{-1}$. Finally, to each (anti-)chiral external leg at zero momentum is assigned a factor of the (anti-)chiral background field $\delta M^{au}(x, \theta)$ ($\delta\overline{M}^{au}(x, \overline{\theta})$) all at the same x. Overall momentum conservation means that the diagram has a factor of $\int d^4x$. The $\delta^4(\theta - \theta')$ for each internal propagator together with the $\int d^4\theta$ integrals at each vertex leave just one overall $\int d^4\theta$ for the diagram.

Before going on to the cases where the effective superpotentials are singular, we start by first looking at the $n_f = n_c + 1$ and $n_f = n_c + 2$ cases. These cases have regular superpotentials and are simple enough to show the details of the calculations. Although the superpotentials are regular in these cases we nevertheless expand them around the modified vacuum (3.7), and then take the limit $\varepsilon \to 0$ at the end. The purpose of doing the calculations around M^ε_0 (rather than M_0) is to familiarize the reader with how the calculations will be implemented for singular superpotentials where expanding around the modified vacuum is necessary.

3.3 $n_f = n_c + 1$

This case is special since the superpotential is of a different form (2.2), involving the Lagrange multiplier field Σ. Expanding (2.2) around M^ε_0, we have

$$W^\varepsilon_{\text{eff}} = [\text{Pf}\, M^\varepsilon_0 - \Lambda^{2(n_c+1)}]\ \delta\Sigma + [(\text{Pf}\, M^\varepsilon_0)(M^\varepsilon_0)^{-1}_{ji}]\ \delta M^{ij} \delta\Sigma$$

$$
\begin{aligned}
&+ (\mathrm{Pf}\, M_0^\varepsilon)\Big[(M_0^\varepsilon)^{-1}_{\ell k}(M_0^\varepsilon)^{-1}_{ji} + (M_0^\varepsilon)^{-1}_{j\ell}(M_0^\varepsilon)^{-1}_{ki} \\
&\qquad\qquad + (M_0^\varepsilon)^{-1}_{jk}(M_0^\varepsilon)^{-1}_{i\ell}\Big]\delta M^{ij}\delta M^{k\ell}\delta\Sigma + \cdots \\
&= [\mathrm{Pf}\, M_0^\varepsilon - \Lambda^{2(n_c+1)}]\ \delta\Sigma - \mu^{n_c}\epsilon_{uv}\ \delta M^{uv}\delta\Sigma - \mu^{n_c-1}\epsilon_{uv}J_{ab}\ \delta M^{au}\delta M^{bv}\delta\Sigma + \cdots,
\end{aligned}
$$

where we have just expressed the terms which are relevant in reproducing the multi-fermion F-term for $n_f = n_c + 1$. Since the superpotential includes the additional field Σ, we cannot use the coefficients for various superspace Feynman diagrams as expressed in (3.10) and (3.13). Instead, reading the appropriate terms off the $W^\varepsilon_{\text{eff}}$ expansion, the propagator between $\delta\Sigma$ and δM^{uv} is accompanied by a factor of $\epsilon^{uv}/[p^2 + (\mu\overline{\mu})^{n_c}]$, a vertex of $\delta M^{au}\delta M^{bv}\delta\Sigma$ comes with a factor of $\epsilon^{uv}J^{ab}\mu^{n_c-1}$, and the $\delta\Sigma$ vertex with a factor of $(\mathrm{Pf}\, M_0^\varepsilon - \Lambda^{2(n_c+1)})$. Evaluating the diagram in figure 3, we have

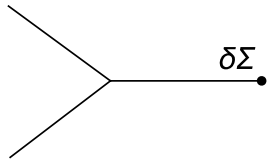


Figure 3: The diagram reproducing the F-term for $n_f = n_c + 1$ flavors.

$$
\delta S \sim \int d^4x\ d^4\theta \left(\frac{\mathrm{Pf}\, M_0^\varepsilon - \Lambda^{2(n_c+1)}}{\overline{\mu}\mu^{n_c-1}}\right)\epsilon^{uv}J^{ab}\,\delta\overline{M}_{au}\cdot\delta\overline{M}_{bv}. \tag{3.14}
$$

In the $\varepsilon \to 0$ limit, $\mathrm{Pf}\, M_0^\varepsilon$ vanishes leaving us with

$$
\delta S \sim \int d^4x\ d^2\theta\, \frac{1}{\overline{\mu}\mu^{n_c-1}}\ \epsilon^{uv}J^{ab}\,(\overline{D}\delta\overline{M}_{au}\cdot\overline{D}\delta\overline{M}_{bv}), \tag{3.15}
$$

where we have traded a $\int d^2\overline{\theta}$ for a $\overline{D}^2$ and used the equation of motion $\overline{D}^2\delta\overline{M} = 0$ to leading order in $\delta\overline{M}$ to distribute the $\overline{D}$'s amongst $\delta\overline{M}$'s. The above expression for δS is the higher-derivative F-term for $n_f = n_c + 1$.

3.4 $n_f = n_c + 2$

In order to reproduce the F-term for $n_f = n_c + 2$ flavors we need four massless anti-chiral legs. There are only two such diagrams, shown in figure 4. Diagram (a) with an amputated 4-vertex ($m = l = 4$) does not have the right structure to be an F-term because, in the $\varepsilon \to 0$ limit, it contributes to the action the term

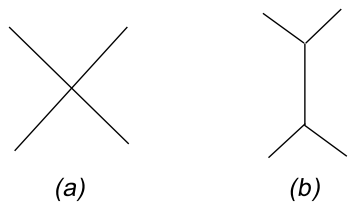


Figure 4: Diagrams with four massless external anti-chiral legs for $n_f = n_c + 2$. (a) The amputated 4-vertex which does not have the right structure. (b) This diagram reproduces the multi-fermion F-term.

$$
\begin{aligned}
\int d^4x\ d^4\theta\, \frac{\overline{\lambda}}{\overline{\mu}^{2-n_c}}\ A^{abcd}_{a'b'c'd'}\ \epsilon^{uvwx}J^{a'b'}J^{c'd'} \\
\times\, \delta\overline{M}_{au}\delta\overline{M}_{bv}\delta\overline{M}_{cw}\delta\overline{M}_{dx},
\end{aligned} \tag{3.16}
$$

where $A^{abcd}_{a'b'c'd'}$ is a non-vanishing tensor which determines how $ab\cdots$ indices are contracted with $a'b'\cdots$ indices. Even if we traded a $\int d^2\overline{\theta}$ for a $\overline{D}^2$ and distributed the $\overline{D}$'s among $\delta\overline{M}_{au}$'s, we would still need another $\overline{D}^2$. Also, the coefficient in the integrand of (3.16) does not match that of (3.4) for $n_f = n_c + 2$. This term is probably just a correction to the Kähler potential, though we have not ruled out the possibility that it is a new global F-term different from (3.6).

Diagram (b) consists of two external anti-chiral vertices and one anti-chiral propagator, and gives

$$\begin{aligned}\delta S \sim \int d^4x\ d^4\theta_1 d^4\theta_2\ \delta\overline{M}_{au}(\theta_1)\delta\overline{M}_{bv}(\theta_1)J^{ab}(J^{us}J^{vt}-J^{ut}J^{vs})\frac{\overline{\lambda}}{\overline{\varepsilon}^{\gamma_{3,2}}\overline{\mu}^{\kappa_2}} \\ \times\ \delta^4(\theta_1-\theta_2)(J_{sp}J_{tq}-J_{sq}J_{tp})\frac{\overline{\varepsilon}^{\alpha}}{\overline{\lambda}\overline{\mu}^{\beta}}\frac{\overline{D}^2}{p^2}\left(1+\left|\frac{\varepsilon^{\alpha}}{\lambda\mu^{\beta}}\right|^2 p^2\right)^{-1} \\ \times\ \frac{\overline{\lambda}}{\overline{\varepsilon}^{\gamma_{3,2}}\overline{\mu}^{\kappa_2}}(J^{wp}J^{xq}-J^{wq}J^{xp})J^{cd}\ \delta\overline{M}_{cw}(\theta_2)\delta\overline{M}_{dx}(\theta_2). \qquad (3.17)\end{aligned}$$

Using the values $\alpha=0$, $\beta=n_c$, $\gamma_{3,2}=0$ and $\kappa_2=1-n_c$, and substituting them in (3.17), we obtain

$$\begin{aligned}\delta S &\sim \int d^4x\ d^4\theta\,\delta\overline{M}_{au}\delta\overline{M}_{bv}\frac{\overline{\lambda}}{\overline{\mu}^{2-n_c}}\frac{\overline{D}^2}{p^2}\left[1-|\lambda\mu^{n_c}|^{-2}p^2+\mathcal{O}(p^4)\right]\delta\overline{M}_{cw}\delta\overline{M}_{dx} \\ &= \frac{\overline{\lambda}}{\overline{\mu}^{2-n_c}}\ \epsilon^{uvwx}J^{ab}J^{cd}\int d^4x\ d^2\overline{\theta}\ \delta\overline{M}_{au}\delta\overline{M}_{bv}\frac{D^2\overline{D}^2}{p^2}(\delta\overline{M}_{cw}\delta\overline{M}_{dx}) \\ &\quad -\frac{\epsilon^{uvwx}J^{ab}J^{cd}}{\lambda\mu^{n_c}\overline{\mu}^2}\int d^4x\ d^2\theta\ \overline{D}^2\left[\delta\overline{M}_{au}\delta\overline{M}_{bv}\overline{D}^2(\delta\overline{M}_{cw}\delta\overline{M}_{dx})\right]+\mathcal{O}(p^2) \\ &= \frac{\overline{\lambda}}{\overline{\mu}^{2-n_c}}\ \epsilon^{uvwx}\int d^4x\ d^2\overline{\theta}\ \delta\overline{M}_{au}\delta\overline{M}_{bv}\delta\overline{M}_{cw}\delta\overline{M}_{dx} \\ &\quad -\frac{\epsilon^{uvwx}}{\lambda\mu^{n_c}\overline{\mu}^2}\int d^4x\ d^2\theta\ (\overline{D}\delta\overline{M}_{au}\cdot\overline{D}\delta\overline{M}_{bv})(\overline{D}\delta\overline{M}_{cw}\cdot\overline{D}\delta\overline{M}_{dx})+\mathcal{O}(p^2), \qquad (3.18)\end{aligned}$$

where in the second line in (3.18) we have traded an $\int d^2\theta$ for a D^2 and used the identity $D^2\overline{D}^2=p^2$ on antichiral fields to cancel the IR pole. We then traded an $\int d^2\overline{\theta}$ for a $\overline{D}^2$ in the third line and used the equation of motion $\overline{D}^2\delta\overline{M}=\mathcal{O}(\delta\overline{\mathcal{M}})$ to distribute the $\overline{D}$'s in the fifth line. In the last two lines, the first term in the expansion does not have the right structure the multi-fermion F-term, but the second term is, up to some numerical factor, the multi-fermion F-term in (3.4) for $n_f=n_c+2$. All other diagrams in the expansion vanish in the limit $p\to 0$.

3.5 $n_f=n_c+3$

This is the first case where we have a singular superpotential. The F-term for $n_f=n_c+3$ has six external anti-chiral massless legs so we have to look for those Feynman diagrams with only six external anti-chiral legs. There are five different possibilities (plus their crossed-channels), see figure 5. Among these diagrams, The four diagrams in figure 5(a) either do not have the right structure to be a multi-fermion F-term, or have zero coefficient. For example, the second graph from the left in figure 5(a) comes with zero coefficient because it has vertices with an odd number of massless legs. The rest of diagrams in figure 5(a) are probably corrections to the Kähler term, though we have not ruled out the possibility that some of them might contribute to new classes of global F-terms different from (3.6).

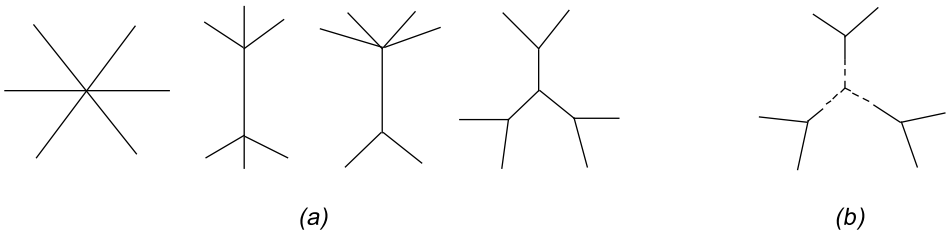


Figure 5: Diagrams with six massless external anti-chiral legs for $n_f = n_c + 3$. (a) Diagrams which do not have the right structure. (b) The only diagram contributing to the F-term (3.4).

The only diagram with the right structure is 5(b): three external anti-chiral vertices with one chiral internal vertex. Evaluating this diagram gives

$$\begin{aligned}\delta S \sim \int d^4x\, d^4\theta B^{uvwxyz}_{u'v'w'x'y'z'} J^{u'v'} J^{w'x'} J^{y'z'} J^{ab} \delta\overline{M}_{au} \delta\overline{M}_{bv} \frac{\overline{\lambda}}{\overline{\varepsilon}^{\gamma_{3,2}} \overline{\mu}^{\kappa_2}} \left|\frac{\varepsilon^\alpha}{\lambda\mu^\beta}\right|^2 \left(1 + \left|\frac{\varepsilon^\alpha}{\lambda\mu^\beta}\right|^2 p^2\right)^{-1} \\ \times \frac{\lambda}{\varepsilon^{\gamma_{3,0}} \mu^{\kappa_0}} \left|\frac{\varepsilon^\alpha}{\lambda\mu^\beta}\right|^2 \left(1 + \left|\frac{\varepsilon^\alpha}{\lambda\mu^\beta}\right|^2 p^2\right)^{-1} \frac{\overline{\lambda}}{\overline{\varepsilon}^{\gamma_{3,2}} \overline{\mu}^{\kappa_2}} J^{cd}\, \overline{D}^2 (\delta\overline{M}_{cw} \delta\overline{M}_{dx}) \\ \times \left|\frac{\varepsilon^\alpha}{\lambda\mu^\beta}\right|^2 \left(1 + \left|\frac{\varepsilon^\alpha}{\lambda\mu^\beta}\right|^2 p^2\right)^{-1} \frac{\overline{\lambda}}{\overline{\varepsilon}^{\gamma_{3,2}} \overline{\mu}^{\kappa_2}} J^{ef}\, \overline{D}^2 (\delta\overline{M}_{ey} \delta\overline{M}_{fz}),\end{aligned} \tag{3.19}$$

with $B^{uvwxyz}_{u'v'w'x'y'z'}$ being a tensor contracting $uv\cdots$ indices to $u'v'\cdots$ indices. Substituting the values $\alpha = \frac{1}{2}$, $\beta = \frac{n_c}{2}$, $\kappa_0 = -\frac{n_c}{2}$, $\kappa_2 = 1 - \frac{n_c}{2}$, $\gamma_{3,0} = \frac{3}{2}$ and $\gamma_{3,2} = \frac{1}{2}$ into (3.19) and taking the limit $\varepsilon \to 0$, we obtain

$$\begin{aligned}\delta S \sim \int d^4x\, d^2\theta\ \frac{\epsilon^{uvwxyz}}{\lambda^2 \overline{\mu}^3 \mu^{n_c}}\ J^{ab} J^{cd} J^{ef} (\overline{D}\delta\overline{M}_{au} \cdot \overline{D}\delta\overline{M}_{bv}) \\ (\overline{D}\delta\overline{M}_{cw} \cdot \overline{D}\delta\overline{M}_{dx})(\overline{D}\delta\overline{M}_{ey} \cdot \overline{D}\delta\overline{M}_{fz}),\end{aligned} \tag{3.20}$$

where we have used the fact that in the $\varepsilon \to 0$ limit the flavor symmetry group is enhanced to $\mathrm{Sp}(2n_c) \times \mathrm{SU}(2n_f - 2n_c)$. This expression coincides with (3.4) for $n_f = n_c + 3$. Since this was the only diagram contributing in the $n_f = n_c + 3$ case, there can be no cancellation of its coefficient. This shows that the $n_f = n_c + 3$ singular superpotential indeed reproduces the corresponding higher-derivative global F-term in perturbation theory.

3.6 $n_f \geq n_c + 4$

As we go higher in the number of flavors, however, the number of diagrams contributing to each amplitude rapidly increases. For instance, just among the class of internally purely-chiral diagrams illustrated in figure 6, there are are four superspace Feynman diagrams in the case of $n_f = n_c + 4$ flavors each with the right structure to contribute to (3.4). But since now multiple diagrams contribute, we must show in addition that no cancellations occur that could set the coefficient of the higher-derivative term to zero. This seems quite complicated, as it depends on the signs and tensor structures of the vertices. Some sort of symmetry argument is clearly wanted, but still eludes us.

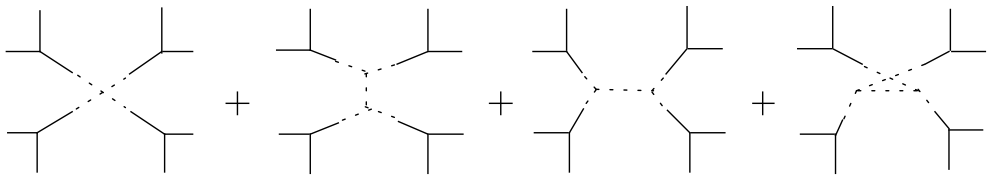


Figure 6: Diagrams which have the right structure to give a higher-derivative F-term for $n_f = n_c + 4$.

In addition, there are now also other classes of diagrams which are neither purely anti-chiral (as in figure 5(a)) or internally purely chiral (as in figure 6). It is not clear whether these mixed diagrams will also contribute to higher-derivative amplitudes of the form (3.4) or not.

Acknowledgments

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